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Thus, for $n=5$, we have

$$1.2.3.4.5=5^5-5(4)^5+10(3)^5-10(2)^5+5(1)^5\ldots\ldots(11).$$

Some new and interesting properties of prime numbers.

If $n+1$ is a prime number, then

$$(a+nb)^x+[a+(n-1)b]^x+[a+(n-2)b]^x+\ldots\ldots(a+b)^x+a^x=m(n+1)\ldots\ldots(12),$$

where m is an integer and x any integer less than n.

In (12) for $a=0$ and $b=1$, we have

$$n^x+(n-1)^x+(n-2)^x\ldots\ldots2^x+1=m(n+1)\ldots\ldots(13).$$

Thus, 11 will exactly divide

$$10^x+9^x+8^x+7^x+6^x+5^x+4^x+3^x+2^x+1\ldots\ldots(14)\text{ where }x=9,8,7,6,5,4,3,2,1.$$

The converse of formulas (12) or (13) is not always true, but the following are true only when $n+1$ is a prime number.

$$(a+n)^n+(a+n-1)^n+(a+n-2)^n+\ldots\ldots a^n+1=m(n+1)\ldots\ldots(15).$$

Making $a=0$, we have

$$n^n+(n-1)^n+(n-2)^n\ldots\ldots1^n+1=m(n+1)\ldots\ldots(16).$$

That is, S_n+1 is divisible by $n+1$ when it is a prime number and only when it is prime. So far as I know this furnishes an entirely *new criterion* of *prime numbers.*

NOTE. The preceding formulas are taken from a paper, by the author, on "The nth power of any number expressed as the sum of the nth powers of other numbers, n being any positive integer;" which was read before the New York Mathematical Society, Dec. 3d, 1892.

NON-EUCLIDEAN GEOMETRY: HISTORICAL AND EXPOSITORY.

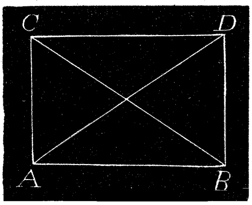
BY GEORGE BRUCE HALSTED, A. M., (Princeton) Ph. D., (Johns Hopkins); Member of the London Mathematical Society; and Professor of Mathematics in the University of Texas, Austin, Texas.

CHAPTER SECOND.

THE FIRST TREATISE ON NON-EUCLIDEAN GEOMETRY.

[Continued from the May Number.]

PROPOSITION I. *If two equal straights* [sects] (*fig* 1.) AC, BD, *make with the straight AB angles equal toward the same parts: I say that the angles at the join CD will be mutually equal.*

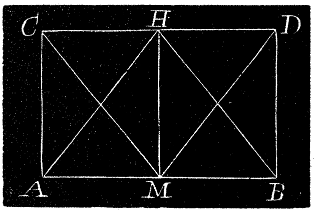


PROOF. Join AD, CB. Then consider the triangles CAB, DBA. It follows (Eu. I. 4.) that the bases CB, AD will be equal.

Then consider the triangles ACD, BDC. It

follows (Eu. I. 8.) that the angles ACD, BDC will be equal. Quod erate demonstrandum.

PROPOSITION II. *Retaining the uniform quadrilateral $ABCD$, bisect the sides AB, CD (fig. 2) in the points M and H. I say the angles at the join MH will then be right.*



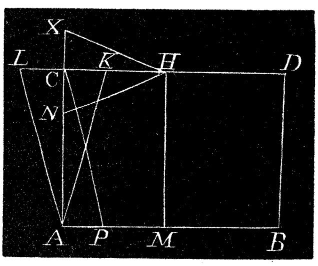
PROOF. Join AH, BH, and likewise CM, DM.

Because in this quadrilateral the angles A, and B are taken equal and likewise (from the preceding proposition) the angles C, and D are equal; it follows (Eu. I. 4.) (noting the equality of the sides) that in the triangles CAM, DBM, the bases CM, DM will be equal; and likewise, in the triangles ACH, BDH, the bases AH, BH.

Therefore; comparing the triangles CHM, DHM, and in turn the triangles AMH, BMH; it follows (Eu. I. 8.) that we have mutually equal, and therefore right the angles at the points M, and H.

Quod erat demonstrandum.

PROPOSITION III. *If two equal straights* [sects] (*fig* 3.) AC, BD *stand perpendicular to any straight AB: I say the join CD will be equal, or less, or greater than that AB, according as the angles at the same CD are right, or obtuse, or acute.*



Proof of the First Part. Each angle C, and D, being right; suppose, if it were possible, either one of those, as DC, greater than the other BA.

Take in DC the piece DK equal to BA, and join AK. Since therefore on BD stand perpendicular the equal straights BA, DK, the angles BAK, DKA will be equal (P. I.). But this is absurd; since the angle BAK is by construction less than the assumed right angle BAC; and the angle DKA is by construction external,and therefore (Eu. I. 16.) greater than the internal and opposite DCA, which is supposed right. Therefore neither of the aforesaid straights, DC, BA, is greater than the other, whilst the angles at the join CD are right; and therefore they are mutually equal.

Quod erat primo loco demonstrandum.

PROOF OF THE SECOND PART. But if the angles at the join CD are obtuse bisect AB, and CD, in the points M, and H, and, join MH.

Since therefore on the straight MH stand perpendicular (P. II.) the two straights AM, CH, and at the join AC is a right angle at A, the straight CH will not be (P. I.) equal to this AM, since a right angle is lacking at C.

But neither will it be greater: otherwise in HC the piece KH being assumed equal to this AM, the angles at the join AK will be (P. I.) equal.

But this is absurd, as above. For the angle MAK is less than a right; and the angle HKA is (Eu. I. 16.) greater than an obtuse, such as the internal and opposite HCA is supposed.

It remains therefore, that CH, whilst the angles at the join CD are taken obtuse, is less than this AM; and therefore CD double the former is less than AB double the latter. Quod erat secondo loco demonstrandum.

PROOF OF THE THIRD PART. Finally however, if the angles at the join CD are acute, MH being constructed as before perpendicular (P. II.), we proceed thus. Since on the straight MH stand perpendicular two straights AM, CH, and at the join AC is a right angle at A, (as above) the straight CH will not be equal to this AM since the angle at C is not right. But neither will it be less: otherwise; if in HC produced HL is taken equal to this AM; the angles at the join AL will be (as above) equal.

But this is absurd. For the angle MAL is by construction greater than the assumed right MAC; and the angle HLA is by construction internal, and opposite, and therefore less than (Eu. I. 16.) the external HCA, which is assumed acute.

It remains therefore, that CH, whilst the angles at the join CD are acute, is greater than this AM, and therefore CD the double of the former is greater than AB the double of the latter. Quod erat tertio loco demonstrandum.

Therefore it is established that the join CD will be equal, or less, or greater than this AB, according as the angles at the same CD are right, or obtuse, or acute. Quae erant demonstranda.

COROLLARY I. Hence in every quadrilateral containing assuredly three right angles, and one obtuse, or acute, the sides adjacent to this oblique angle are less than the opposite sides, if this angle is obtuse, but greater if it is acute.

For this has just now been demonstrated of the side CH relatively to the opposite side AM; in the same way it is demonstrated of the side AC relatively to the opposite side MH. For since the straights AC, MH, are perpendicular to this AM, they cannot (P. I.) be mutually equal, on account of the unequal angles at the join CH.

But neither (in the hypothesis of an obtuse angle at C) can a certain AN, a piece of this AC, than which certainly the aforesaid AC is greater, be equal to this MH: otherwise (P. I.) the angles at the join HN would be equal; which is absurd, as above.

Again however (in the hypothesis of an acute angle at this point C) if you take a certain AX, assumed on AC produced, than which certainly the just mentioned AC is less, equal to this MH; now by this same title the angles at HX will be equal; which assuredly is absurd in the same way, as above.

It remains therefore, that indeed in the hypothesis of an obtuse angle at this point C, the side AC is less than the opposite side MH; but in the hypothesis of an acute angle is greater than it. Quod erat intentum.

COROLLARY II. But by much more will CH be greater than any piece of this AM, as for instance PM, with which of course the join CP makes an angle still more acute with this CH towards the parts of the point H, and obtuse (Eu. I. 16.) with this PM towards the parts of the point M.

COROLLARY III. Again it abides that all things aforesaid equally result, whether the assumed perpendiculars AC, and BD are of some length

fixed by us, or are, or are supposed infinitesimal.

This indeed ought opportunely to be noted in remaining subsequent propositions.

PROPOSITION IV. *But inversely (the figure of the preceding proposition remaining) the angles at the join CD will be right, or obtuse, or acute, according as the straight CD is equal, or less, or greater than the opposite AB.*

PROOF. For if the straight CD is equal to the opposite AB, and nevertheless the angles at it are either obtuse, or acute; now these such angles prove it (P. III.) not equal, but less, or greater that the opposite AB; which is absurd against the hypothesis.

The same uniformly avails in regard to the remaining cases. It holds therefore that the angles at the join CD are either right, or obtuse, or acute, according as the straight CD is equal, or less, or greater than the opposite AB. Quod erat demonstrandum.

DEFINITIONS. Since (P. I.) the straight joining the extremities of equal perpendiculars standing upon the same straight (which we call base), makes equal angles with these perpendiculars; therefore there are three hypotheses to be distinguished about the species of these angles. And the first indeed I will call hypothesis of right angle; the second however, and the third I will call hypothesis of obtuse angle, and hypothesis of acute angle.

ARITHMETIC.

Conducted by B.F.FINKEL, Kidder, Missouri. All Contributions to this department should be sent to him.

SOLUTIONS TO PROBLEMS.

16. Proposed by EARL D. WEST, West Middleburg, Logan County, Ohio.

How many stakes can be driven down upon a space 15 feet square allowing no two to be nearer each other than $1\frac{1}{2}$ feet, and how many allowing no two to be nearer than $1\frac{1}{4}$ feet?

Solution by B. F. FINKEL, A. M., Professor of Mathematics, Kidder Institute, Kidder, Missouri.

(*a*) 1. Since the least distance from one stake to another is $1\frac{1}{2}$ ft., the number of $1\frac{1}{2}$ ft. spaces in the base line AB is 15 ft. $\div 1\frac{1}{2}$ft. or 10. Hence, we can place 11 stakes on the base line AB, and, by *square arrangement*, we can place on the square $ABCD$ 11 rows with 11 stakes in a row, in all 11×11 stakes or 121 stakes.

2. By *quincunx arrangement*, we can place 11 stakes on the base line AB and over these, as vertices of equilateral triangles 10 stakes. Now the width Bi of the strip $ABil$ is $\sqrt{oB^2-oi^2}=\sqrt{(1\frac{1}{2})^2-(\frac{3}{4})^2}=\frac{3}{4}\sqrt{3}$ ft. $=1.2990381+$ ft. Hence, the width of the strip le is 1.5 ft. $-1.2990381+$ ft. $=2009619-$ ft.